Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

1366780805

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **October/November 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point on a smooth horizontal surface. The particle is moving in a circular path on the surface. The tension in the string is 20 N.

Find how many revolutions the particle makes per minute. [3]

2 A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is fixed to a point O on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected along the surface in the direction OP. When the length of the string is $\frac{5}{4}a$, the speed of P is v. When the length of the string is $\frac{3}{2}a$, the speed of P is $\frac{1}{2}v$.

(a) Find an expression for v in terms of a and g. [4]

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(b) Find, in terms of g, the acceleration of P when the stretched length of the string is $\frac{3}{2}a$. [2]

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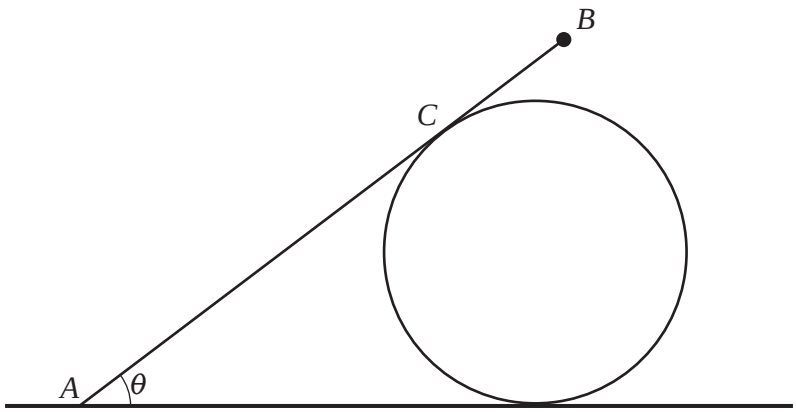

A smooth cylinder is fixed to a rough horizontal surface with its axis of symmetry horizontal. A uniform rod AB, of length $4a$ and weight W, rests against the surface of the cylinder. The end A of the rod is in contact with the horizontal surface. The vertical plane containing the rod AB is perpendicular to the axis of the cylinder. The point of contact between the rod and the cylinder is C, where $AC = 3a$. The angle between the rod and the horizontal surface is θ where $\tan\theta = \frac{3}{4}$ (see diagram). The coefficient of friction between the rod and the horizontal surface is $\frac{6}{7}$.

A particle of weight kW is attached to the rod at B. The rod is about to slip. The normal reaction between the rod and the cylinder is N.

(a) Show that $N = \frac{8}{15}W(1+2k)$. [2]

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(b) Find the value of k. [5]

4 A particle of mass 0.5 kg moves along a horizontal straight line. Its velocity is $v\,\mathrm{m\,s^{-1}}$ at time t s. The forces acting on the particle are a driving force of magnitude 50 N and a resistance of magnitude $2v^2$ N. The initial velocity of the particle is $3\,\mathrm{m\,s^{-1}}$.

(a) Find an expression for v in terms of t. [7]

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(b) Deduce the limiting value of v. [1]

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5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The string is held taut with OP horizontal. The particle P is projected vertically downwards with speed $\sqrt{\frac{1}{3}ag}$ and starts to move in a vertical circle. P passes through the lowest point of the circle and reaches the point Q where OQ makes an angle θ with the downward vertical. At Q the speed of P is $\sqrt{kag}$ and the tension in the string is $\frac{11}{6}mg$.

(a) Find the value of k and the value of $\cos\theta$. [4]

At Q the particle P becomes detached from the string.

(b) In the subsequent motion, find the greatest height reached by P above the level of the lowest point of the circle. [4]

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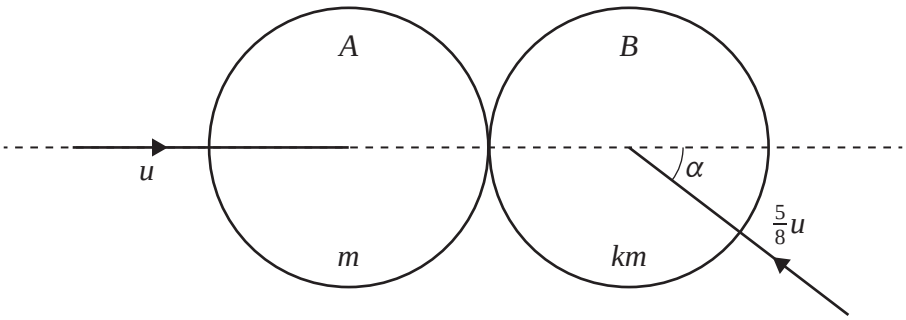

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are moving on a horizontal surface with speeds u and $\frac{5}{8}u$ respectively. Immediately before the spheres collide, A is travelling along the line of centres, and B's direction of motion makes an angle α with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$.

After the collision, the direction of motion of B is perpendicular to the line of centres.

(a) Find the value of k. [4]

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(b) Find the loss in the total kinetic energy as a result of the collision. [4]

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7 A particle P is projected with speed $V\,\mathrm{m\,s^{-1}}$ at an angle 75° above the horizontal from a point O on a horizontal plane. It then moves freely under gravity.

(a) Show that the total time of flight, in seconds, is $\frac{2V}{g}\sin 75°$. [2]

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A smooth vertical barrier is now inserted with its lower end on the plane at a distance 15 m from O. The particle is projected as before but now strikes the barrier, rebounds and returns to O. The coefficient of restitution between the barrier and the particle is $\frac{3}{5}$.

(b) Explain why the total time of flight is unchanged. [1]

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(c) Find an expression for V in terms of g. [7]

Additional page

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